1. The length of a shadow of a tree is 130 feet when the angle of elevation to the sun is $\theta$.
   a) Draw a picture and write the height of the tree as a function of $\theta$.
   
   b) If $\theta = 25^\circ$, find the height of the tree.
   
   c) Rewrite your function in part a so that $\theta$ is a function of the height of the tree.
   
   d) If the height of the tree is 100 feet, what is the angle of elevation?

2. The sonar of a navy cruiser detects a nuclear submarine that is 4000 feet from the cruiser. The angle between the water level and the submarine is $31.5^\circ$. How deep is the submarine?

3. An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are $4^\circ$ and $6.5^\circ$. How far apart are the ships?
4. Two fire towers are 30 kilometers apart, tower $A$ being due west of tower $B$. A fire is spotted from the towers, and the bearings from $A$ and $B$ are E $14^\circ$ N and W $34^\circ$ N, respectively. Find the distance $d$ of the fire from the line segment $AB$.

5. A shoreline runs north-south, and a boat is due east of the shoreline. The bearings of the boat from two points on the shore are $110^\circ$ and $100^\circ$. Assume the points are 550 feet apart. How far is the boat from the shore?

6. The horizontal distance that a projectile will travel in the air is given by the equation $R = \frac{\left(v_0\right)^2 \sin(2\theta)}{g}$, where $v_0$ is the initial velocity of the projectile, $\theta$ is the angle of elevation, and $g$ is acceleration due to gravity. For this problem, let’s assume $g = 9.8 \text{ m/s}^2$.

   a) Suppose you are able to throw the ball with an initial velocity of 40 m/s. How much further will the ball travel, if the angle of elevation is changed from $20^\circ$ to $25^\circ$?

   b) If you can throw a baseball with an initial speed of 34.8 m/s, at what angle of elevation should you direct the throw so that the ball travels a distance of 107 meters before striking the ground?
1. The following angles are given to you in radian measure. Without converting to degrees, draw a sketch of each angle in standard position AND give the reference angle.

   a) \( \frac{8\pi}{5} \)  
   b) \( \frac{\pi}{6} \)  
   c) \( \frac{13\pi}{9} \) 

   d) \( \frac{3\pi}{2} \)  
   e) 5.6  
   f) 1.3

2. If you are given a degree measurement, how do you convert it to radians?

3. If you are given a radian measurement, how do you convert it to degrees?

4. Convert the following angle measurements from degrees to radians (or vice-versa).

   a) \( \frac{2\pi}{3} \)  
   b) 24°

   c) 4.6 
   d) 310°

5. What is the formula for finding arc length? What does each letter represent?

6. A sector has an arc length 12 cm and a central angle of \( \frac{\pi}{3} \) radians. Find the radius of the circle.
7. The skateboard ramp at the right is called a quarter pipe. The curved surface is determined by the radius of a circle. Find the length of the curved part of the ramp.

8. The paddlewheel of a riverboat has a diameter of 24 feet. Find the arc length of the circle made when the paddlewheel rotates 300°.

9. How many degrees are in one revolution? How many degrees in \( \frac{1}{8} \) of a revolution?

10. How many radians are in one revolution? How many radians in \( \frac{1}{3} \) of a revolution?

11. The radius of a car wheel is 13 inches. How many revolutions per minute is the wheel making when the car is traveling at 40 mph.

12. A speedometer on a car is calibrated for the specific tire size for which the car was built. The speedometer uses the revolutions per minute of the tires to calculate the speed. If the same car in the last question is “tricked out” to have 15 inch wheels and its wheels are turning the same revolutions per minute as you found above, now how fast is the car actually traveling in mph?

13. Monster Truck tires have a radius of 33 inches. How far does a monster truck travel in feet after just three-fourths of a tire rotation?

14. A radial arm saw has a circular cutting blade with a diameter of 10 inches. It spins at 2000 rpm (revolutions per minute). If there are 12 cutting teeth per inch on the cutting blade, how many teeth cross the cutting surface each second?
There are two other ways to measure angles not discussed in the notes. The first is Degrees, Minutes, Seconds (DMS). Map coordinates are given using latitude and longitude in terms of degrees, minutes, seconds or in terms of a decimal form of the degree. We use the minutes and seconds notation to replace the decimal points on a degree.

We denote 53 degrees, 20 minutes, and 19 seconds as $53^\circ20'19''$.

You can use your TI-83 calculator to convert from DMS to degrees and vice-versa.

To convert from DMS to degrees, simply type in the symbols for degrees ($2^{nd}$ APPS, 1), minutes ($2^{nd}$ APPS, 2), and seconds (ALPHA, +) and pressing ENTER.

15. Convert $53^\circ20'19''$ to degrees to the nearest thousandth.

To convert from degrees to DMS, simply enter the degree in your calculator followed by $\downarrow$DMS ($2^{nd}$ APPS, 4)

16. Convert $34.843^\circ$ to DMS.

Another way to measure angles is bearings. In navigation, the course or bearing of an object is sometimes given as the angle of the line of travel measured clockwise from due north. In other words, the initial side of a bearing is north.

17. Find the angle in degrees that describes the compass bearing.
   a) Bearing “SE”
   b) Bearing “WNW”

18. Find the compass bearing closest to each angle.
   a) Bearing $145^\circ$
   b) Bearing $318^\circ$

19. The captain of the tourist boat $Julia$ out of Oak Harbor follows a bearing of $38^\circ$ for 2 miles and then changes to a bearing of $47^\circ$ course for the next 4 miles. Draw a sketch of this trip.

20. Convert a bearing of $140^\circ$ into a radian measurement.
1. Find the six trigonometric ratios of angle \( A \) in the triangle below.

In questions 2 and 3, assume that the given angle is an acute angle in a right triangle satisfying the given conditions. Evaluate the other 5 trigonometric functions.

2. \( \cot \theta = \frac{11}{3} \)

3. \( \csc \theta = \frac{12}{5} \)

For questions 4 – 9, evaluate each without using a calculator.

4. \( \sin \left( \frac{\pi}{3} \right) \)

5. \( \tan \left( \frac{\pi}{4} \right) \)

6. \( \cot \left( \frac{\pi}{6} \right) \)

7. \( \sec \left( \frac{\pi}{3} \right) \)

8. \( \cos \left( \frac{\pi}{4} \right) \)

9. \( \csc \left( \frac{\pi}{3} \right) \)

10. If \( \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \) and \( \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \), what is \( \frac{\sin \theta}{\cos \theta} \)?
11. Complete the following table using the two right triangles and the result from the last question.
…YOU NEED TO MEMORIZE THESE VALUES BEFORE NEXT CLASS!
(more on this at the optional question at the end)

<table>
<thead>
<tr>
<th>( \theta \rightarrow )</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin ( \theta )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cos ( \theta )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tan ( \theta )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. [No Calculator] A tree casts a shadow approximately 24 feet long when the sun’s rays form a 30° angles with the earth. How tall is the tree?

13. [No Calculator] A rope tied to the top of a tent pole on one end and nailed to the ground for support on the other end is 10 feet long and makes a 45° angle with the ground. How tall is the tent pole?

14. [Calculator Required] Solve \( \Delta ABC \) for all of its unknown parts when \( a = 10.3 \) and \( \alpha \) (alpha) = 25°.
15. [Calculator Required] A biologist wants to know the width of a river in order to properly set instruments for studying the pollutants in the water. From point \( A \), the biologist walks downstream 100 feet and sights to point \( C \). From this sighting, it is determined that \( \angle ABC = 58^\circ \). How wide is the river?

16. Find the acute angle \( \theta \) that satisfies each given equation. Give \( \theta \) in both radians and degrees. You should be able to do this WITHOUT the use of a calculator!

a) \( \sin \theta = \frac{1}{2} \)  

b) \( \sin \theta = \frac{\sqrt{2}}{2} \)  

c) \( \cot \theta = \frac{1}{\sqrt{3}} \)

d) \( \cos \theta = \frac{\sqrt{2}}{2} \)  

e) \( \sec \theta = 2 \)  

f) \( \cot \theta = 1 \)

g) \( \tan \theta = \frac{\sqrt{2}}{\sqrt{3}} \)  

h) \( \cos \theta = \frac{\sqrt{2}}{2} \)  

h) \( \tan \theta = 1 \)

17. Using the triangle below, PROVE that if \( \theta \) is an acute angle in any right triangle, then

\[
\left( \sin \theta \right)^2 + \left( \cos \theta \right)^2 = 1.
\]
The chart below can be “memorized” without the triangles if you learn a couple of patterns ...

Step 1: Make sure the chart is setup like it is below
… (30/45/60) and (sine, cosine, tangent)

<table>
<thead>
<tr>
<th>θ →</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin θ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cos θ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tan θ</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 2: Enter 1, 2, and 3, in the first row … and
3, 2, 1 in the second row

Step 3: Take the square root of all the numbers from step 2 …
keep in mind that \( \sqrt{1} = 1 \)

Step 4: Divide ALL 6 of the numbers in the first two rows by 2.

Step 5: Use the fact that \( \tan \theta = \frac{\sin \theta}{\cos \theta} \) to generate the values in the last row.
1. Graph each angle in standard position and give the reference angle. Always give $\theta_{\text{ref}}$ in the same units as the given angle.
   a) $-135^\circ$
   b) $\frac{11\pi}{3}$
   c) $-2$

2. Find one angle that is NOT co-terminal with the others listed.
   a) $150^\circ$, $510^\circ$, $-210^\circ$, $450^\circ$, $870^\circ$
   b) $\frac{5\pi}{3}$, $-\frac{5\pi}{3}$, $\frac{11\pi}{3}$, $-\frac{7\pi}{3}$, $\frac{365\pi}{3}$

3. Find and draw one positive and one negative angle that is co-terminal with the given angle.
   a) $30^\circ$
   b) $-420^\circ$
   d) $\frac{4\pi}{3}$
   f) $-\frac{23\pi}{4}$
4. Point $P$ is on the terminal side of angle $\theta$. Evaluate the six trigonometric functions for $\theta$. If a function is undefined, write undefined.

   a) $P(-5, 12)$
   b) $P(6, -6)$

5. Remembering your 45–45–90 right triangle and the sine and cosine of 45°, explain how you could have predicted the answers for question 4b.

6. Determine the sign (+ or −) of each expression WITHOUT using a calculator.

   a) $\cos 143^\circ$
   b) $\sin \frac{7\pi}{8}$
   c) $\tan 192^\circ$
   d) $\sec \left( -\frac{4\pi}{5} \right)$

7. Find an interval of values on the domain $[0, 2\pi]$ for $\theta$ that makes the following sets of statements true.

   a) $\csc \theta > 0$ and $\cos \theta < 0$
   b) $\tan \theta < 0$ and $\sec \theta > 0$
The next group of questions are designed to help you make some connections about some special theorems from trigonometry WITHOUT actually going through a proof of the theorems…you’re welcome. 😊 They are easiest to approach from a graphical perspective, so we will start there. Finishing this page by next class will earn you some extra credit!

8. Graph any acute radian angle \( \theta \). Then, use that picture to find where each requested angle would be located. Remember, you know how big \( \pi \) radians is in terms of a revolution.

a) Graph \((-\theta)\)  

b) Graph \((\pi + \theta)\)  

c) Graph \((\pi - \theta)\)  

9. Using the graphs above, draw reference triangles to the \( x \) axis. Label the sides of your triangles given that \( \sin \theta = 0.12 \) and \( \cos \theta = 0.05 \) for the original angle \( \theta \). Next, find the value of each expression. Remember All Students Take Classes!!

a) \( \sin (-\theta) = \)  
b) \( \sin (\pi + \theta) = \)  
c) \( \sin (\pi - \theta) = \)  

\( \cos (-\theta) = \)  

\( \cos (\pi + \theta) = \)  

\( \cos (\pi - \theta) = \)  

Let’s see if you “get it”...

10. If \( \cos \theta = -0.4 \), find \( \cos (\pi + \theta) \).

11. If \( \sin \theta = 0.4 \), find \( \sin (-\theta) + \csc \theta \) without a calculator.

One last idea that will be very important next chapter...

12. We know the radius of the Unit Circle is 1 and angle \( \theta \) makes a reference triangle with a right angle. We further know that any missing side of a right triangle can be found with the Pythagorean Theorem and that the sides of the right triangle are \( x \) and \( y \). If you know that \( \sin \theta = y \) and \( \cos \theta = x \), what is the relationship between \( \sin \theta \) and \( \cos \theta \)?

13. Find \( \sin \theta \) when \( \cos \theta = -0.2 \).
Graph each angle and list $\theta_{ref}$. Then, find the exact value of each trigonometric function.

1. $\sin\left(\frac{\pi}{6}\right)$
2. $\sin\left(-\frac{5\pi}{6}\right)$
3. $\sin\left(\frac{11\pi}{6}\right)$
4. $\sin\left(\frac{5\pi}{6}\right)$
5. $\sin\left(\frac{13\pi}{6}\right)$
6. $\sin\left(\frac{7\pi}{6}\right)$

7. What do you notice about the reference angles for questions 1-6? What do you notice about the answers?

8. Do you think a similar pattern would occur if questions 1-6 asked for cosines? For tangents?

9. What is a quadrantal angle? Explain why the 30-60-90 or 45-45-90 right triangles are NOT helpful on quadrantal angles.
Find the exact value of each trigonometric function.

10. \( \cos \left( \frac{2\pi}{3} \right) \)
11. \( \tan(3\pi) \)
12. \( \csc \left( \frac{11\pi}{6} \right) \)

13. \( \sec 135^\circ \)
14. \( \sin(-270^\circ) \)
15. \( \cot(225^\circ) \)

16. \( \sin \left( \frac{7\pi}{4} \right) \)
17. \( \tan \left( -\frac{5\pi}{6} \right) \)
18. \( \sec(-45^\circ) \)

19. \( \sec(-90^\circ) \)
20. \( \tan(270^\circ) \)
21. \( \csc(\pi) \)

22. What do you notice about your answers to questions 19-21? Why does this happen??

23. If \( \cos \theta = -\frac{5}{13} \) and \( \tan \theta > 0 \), find \( \sin \theta \).

24. If \( \cot \theta = -\frac{15}{8} \) and \( \cos \theta < 0 \), find \( \csc \theta \).

25. If \( \sec \theta = \frac{5}{2} \) and \( \sin \theta < 0 \), find \( \cot \theta \).
PreCalculus
Graphing Trig. Functions (Tangents & Reciprocals)

No Calculator

1. On a separate piece of graph paper, graph at least one period of each function below. Be sure to label your axes and clearly identify the asymptotes.

   a) \( y = 4 \tan(3x) \)
   b) \( y = \tan\left(\frac{1}{2}x + \frac{\pi}{4}\right) - 1 \)
   c) \( y = -\tan\left(\frac{\pi}{2}x\right) + 2 \)

For 2 – 10, graph the following trigonometric functions on graph paper. Note the scale on each axis.

2. \( y = \sec(3x) \)
3. \( y = -\csc(4x) \)
4. \( y = 2 \cot(8x) \)
5. \( y = \frac{1}{3}\csc(x) \)
6. \( y = \cot\left(x - \frac{\pi}{2}\right) - 3 \)
7. \( y = -4 \sec\left(\frac{1}{5}x\right) + 2 \)
8. \( y = \sec\left[2\left(x + \frac{\pi}{4}\right)\right] - 2 \)
9. \( y = -\cot\left(\frac{1}{2}x + \frac{\pi}{2}\right) \)
10. \( y = 2 \csc\left(\frac{x}{3} - \frac{\pi}{2}\right) + 1 \)
Pre Calculus
Worksheet: Review of Inverses (prerequisites before 4.7)

1. Choose the best description of a way to think of an “Inverse”

   A. The opposite of something
   B. The reciprocal of something
   C. \( f(g(x)) \)
   D. When we “undo” something

2. How do you find the inverse of an equation? The inverse of a graph?

3. What happens to the Domain and Range of the original function when you find the inverse function?

4. Tell the Domain and Range of the Inverse. Then, find the inverse of each function.
   a) \( k(x) = 4 + \sqrt{2x - 1} \)  
   b) \( m(x) = 7e^{x^2} \)

5. Graph the inverse of the given relation. Tell whether each inverse is a function or is not a function. Explain.
   a.  
   b. \( y = x^2 \)

6. How can you PREDICT whether a function’s inverse will be a function?

7. What did we do first semester to \( y = x^2 \) so that it its inverse was a function?
8. Now, let’s try some inverse trig graphs. You may need to sketch the original trig parent functions with their key points first and then sketch the graph of the inverses.

a. \( y = \sin^{-1}(x) \)  

b. \( y = \cos^{-1}(x) \)  

c. \( y = \tan^{-1}(x) \)  

Think of what we did to \( y = x^2 \) to make the inverse a function...

9. Now highlight a piece of each graph that is a function. The piece you want must include the angles on the interval \((0, \frac{\pi}{2})\) because these angles are in a right triangle. Think about the attributes of the parent function that you should also include. You do NOT have to use the same piece for all three graphs. List an interval for the angles you want to include.

a. \( y = \sin^{-1}(x) \)  

b. \( y = \cos^{-1}(x) \)  

c. \( y = \tan^{-1}(x) \)  

10. Highlight the part of the unit circle corresponding to the angles you selected in question 9.

a. \( y = \sin^{-1}(x) \)  

b. \( y = \cos^{-1}(x) \)  

c. \( y = \tan^{-1}(x) \)  

Finally, let’s think of input and outputs...

11. For the original trig. graphs (\( y = \sin x, y = \cos x, y = \tan x \)), \( x \) gives an ____________ and \( y \) gives a ratio of the sides of a right triangle. For the inverse trig. graphs (\( y = \sin^{-1} x, y = \cos^{-1} x, y = \tan^{-1} x \)), \( x \) gives ________________ and \( y \) gives ________________.
Pre Calculus  
Worksheet 4.7

1. The “input” to a trig function is an _______________, and the “output” is a ______________.

2. Evaluate each of the following trig expressions:
   a) \[ \cos \left( \frac{3\pi}{4} \right) \]
   b) \[ \sin \left( \frac{7\pi}{6} \right) \]
   c) \[ \tan \left( \frac{-5\pi}{3} \right) \]
   d) \[ \sec \left( \frac{5\pi}{2} \right) \]

3. The “input” to an inverse trig function is a _____________, and the “output” is an ________________.

4. For each of the following, what quadrant is the answer in? (You don’t have to find the answer.)
   a) \( \sin^{-1}(0.8) \)
   b) \( \cos^{-1}(0.8) \)
   c) \( \tan^{-1}(0.8) \)

5. Based on your answers to question 4, what conclusion can you make when the “input” of an inverse trig function is a positive value?

6. For each of the following, what quadrant is the answer in? (You don’t have to find the answer.)
   a) \( \sin^{-1}(-0.8) \)
   b) \( \cos^{-1}(-0.8) \)
   c) \( \tan^{-1}(-0.8) \)

7. The following represent EVERY inverse sine function that you are expected to know w/o a calculator.
   a) \( \sin^{-1} \left( \frac{1}{2} \right) \)
   b) \( \arcsin \left( -\frac{1}{2} \right) \)
   c) \( \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) \)
   d) \( \sin^{-1} \left( -\frac{1}{\sqrt{2}} \right) \)
   e) \( \sin^{-1}(-1) \)
   f) \( \arcsin(1) \)
   g) \( \sin^{-1}(0) \)
   h) \( \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \)
   i) \( \sin^{-1} \left( -\frac{\sqrt{3}}{2} \right) \)
8. The following represent EVERY inverse cosine function that you are expected to know w/o a calculator.

a) \(\cos^{-1}\left(\frac{1}{2}\right)\) 

b) \(\arccos\left(-\frac{1}{2}\right)\) 

c) \(\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\) 

d) \(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\) 

e) \(\cos^{-1}\left(-1\right)\) 

f) \(\arccos(1)\)  

g) \(\cos^{-1}(0)\) 

h) \(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\) 

i) \(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\) 

9. The following represent EVERY inverse tangent function that you are expected to know w/o a calculator.

a) \(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\) 

b) \(\arctan\left(-\frac{\sqrt{3}}{3}\right)\) 

c) \(\tan^{-1}\left(\sqrt{3}\right)\) 

d) \(\tan^{-1}\left(-\sqrt{3}\right)\) 

e) \(\tan^{-1}\left(-1\right)\) 

f) \(\arctan(1)\)  

g) \(\tan^{-1}(0)\) 

Find the exact value without a calculator.

10. \(\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right)\)

11. \(\cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right)\)

12. \(\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\)

13. \(\csc\left(\tan^{-1}\left(-1\right)\right)\)
Find an algebraic expression equivalent to the given expression.

14. \( \sin \left( \tan^{-1} x \right) \)  

15. \( \cos \left( \tan^{-1} x \right) \)  

16. \( \tan \left( \arcsin x \right) \)  

17. \( \sin \left( \arccos 3x \right) \)  

For questions 18 and 19, use a calculator to find the approximate value in degrees.

18. \( \arcsin 0.67 \)  

19. \( \tan^{-1} 2.37 \)  

Most calculators do not have keys for evaluating the inverse cotangent, cosecant, and secant functions. The easiest way to do this is to first convert the inverse expression into a normal cotangent, cosecant, or secant function, then use the reciprocal properties and their inverses to evaluate.

While \( \sec(x) = \frac{1}{\cos(x)} \), NOTICE that \( \sec^{-1}(x) \neq \frac{1}{\cos^{-1}(x)} \)  

For questions 20 – 22, evaluate each using your calculator. Be sure you are in radian mode.

20. \( \sec^{-1}(3) \)  

21. \( \cot^{-1}\left(\frac{1}{2}\right) \)  

22. \( \csc^{-1}(-4) \)