Lesson Targets for Intro:
1. Know and be able to explain the definition of a conic section.
2. Identify the general form of a quadratic function (in two variables) as a parabola, circle, ellipse or hyperbola.
3. Determine the type of conic using the discriminant when given the general form equation.
4. Complete the square to write the equation of a conic in transformational form.

**Conic Section:** the shape formed by the intersection of a right circular cone and a plane. The basic conic sections (also called non-degenerative conic sections) are shown below.

- Some textbooks refer to the circle as a specialized form of an ellipse.
- We will study the geometric definitions that make each shape AND the algebraic equation for each shape this unit.
- All conic sections can be modeled by using the general form of a quadratic equation in two variables:
  \[
  Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0
  \]
  where A, B and C are not all zero.

You may recall from previous work with solving quadratic functions, the *discriminant* is the value under the square root of the quadratic formula \((b^2 - 4ac)\). The value of the discriminant tells us the number and type of zeros.

Likewise, the general form for a conic has a discriminant \((B^2 - 4AC)\). The value of the discriminant tells us the type of conic section we have.

\[
\text{If } B^2 - 4AC \text{ is} \begin{cases}  > 0 \\
= 0 \\
< 0 
\end{cases} \quad \text{Remember a circle is a special ellipse where…}
\]

*Example 1:* Use the discriminant to identify the type of conic section.

a) \(3x^2 - 4xy + 2y^2 - 3y = 0\)  

b) \(x^2 - xy = -1\)
In addition to identifying the type of conic section, we will want each equation written in “transformational” form...that is written so that we can easily identify the transformations that have been applied from the parent function.

- To write in change how the equation is written, we ____________________________.
- Notice we may need to ____________________________ for \( x \) or \( y \) or both!

**Example 2:** Use the discriminant to identify the type of conic section. Then, rewrite each general form quadratic into a transformational form.

a) \( x^2 + 4x - 8y + 12 = 0 \)  
b) \( x^2 + y^2 - 6x + 12y + 36 = 0 \)  
c) \( 9x^2 - 4y^2 + 90x - 16y - 115 = 0 \)

**Closing questions...**

Which term from the general form equation is MISSING from all of the equations in example 2?

What do you notice about the equation of the parabola as compared to the other conics for example 2?
8.1 PARABOLAS

Learning Targets for 8.1

1. Write the equation of a parabola when given at least two of the important features of the graph.
2. Identify the key features of a parabola when given an equation.
3. Sketch the graph of a parabola by hand including vertex, directrix, focus and curve.
4. Prove a general form equation is a parabola.

**Parabola:** The set of all points equidistant from a line called the *directrix* and a fixed point called the *focus*.

Vertical Parabolas: \( y - k = \frac{1}{4p} (x - h)^2 \)

Horizontal Parabolas: \( x - h = \frac{1}{4p} (y - k)^2 \)

Notice the vertex is equidistant from the focus and directrix just like every other point on the parabola. This distance is noted by ________________ in the equation above.

Remember: the parabola is ________________ the focus and never crosses the ________________.

**Example 1:** Find the equation for the parabola that satisfies the given conditions. A sketch is helpful!

a) Focus: \((2, -3), \text{ Directrix } x = 5\)  
b) Focus \((-5, 3), \text{ Vertex: } (-5, 6)\)

**Example 2:** Sketch the graph of \((x - 2)^2 = 8(y + 1)\) by hand.
The table below summarizes what we have learned…

<table>
<thead>
<tr>
<th></th>
<th>Vertical</th>
<th>Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Opening</strong></td>
<td>Up: ( p &gt; 0 )</td>
<td>Right: ( p &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>Down: ( p &lt; 0 )</td>
<td>Left: ( p &lt; 0 )</td>
</tr>
<tr>
<td><strong>Equation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vertex</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Focus</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Directrix</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Axis of Symmetry</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Example 3:* Prove that the graph of \( y^2 - 2y + 4x - 11 = 0 \) is a parabola. Then find its vertex, focus, and directrix.
PreCalculus
Exploration: 8.2 Ellipses

An ellipse is different from a circle because it is longer from the center to the edge in one direction than it is in the other direction. Answer the questions below.

1. Examine the ellipses at the right.

   a) What is the same about them?
   
   b) What is different about them?

2. The equation for ellipse A is \( \frac{x^2}{9} + \frac{y^2}{36} = 1 \). The equation for ellipse B is \( \frac{x^2}{36} + \frac{y^2}{9} = 1 \).

   a) How can you determine from the equation whether an ellipse will be longer in the horizontal direction or longer in the vertical direction?

   b) How can you determine the distance from the center of the ellipse to the edge in the horizontal direction?

   c) How can you determine the distance from the center of the ellipse to the edge in the vertical direction?

3. Use the graph of \( \frac{(x-1)^2}{25} + \frac{(y+4)^2}{4} = 1 \) provided at the right to help you answer the questions below.

   a) Based on the translation rules from the equation for a circle and the sample given, how would you translate an ellipse?

   b) Write the equation for the translated form of an ellipse (using variables like x, y, h, k, a and b). Identify the center.
8.2 Ellipses

8.2 ELLIPSE

Learning Targets 8.2
1. Write the equation of an ellipse when given at least two of the important features of the graph.
2. Identify the key features of an ellipse when given an equation.
3. Sketch the graph of an ellipse by hand including center, vertices and co-vertices.
4. Prove a general form equation is an ellipse.

Before watching the Video for Lesson 8.2, complete the Exploration for 8.2!

Ellipse: The set of all points in a plane whose sum of distances from two fixed points is a constant. The fixed points are called ______________. For the equations below, ______________ and ______________.

- Major Axis has length _____ and contains the center, vertices and foci.
- Minor Axis has length _____ is perpendicular to the transverse axis.

Vertical Ellipse: \( \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \)
Horizontal Ellipse: \( \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \)

<table>
<thead>
<tr>
<th>Vertices</th>
<th>Horizontal Ellipse</th>
<th>ALL Ellipses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices</td>
<td>“a” units away from center along major axis.</td>
<td></td>
</tr>
<tr>
<td>Co-Vertices</td>
<td>“b” units away from center along minor axis</td>
<td></td>
</tr>
<tr>
<td>Foci</td>
<td>“c” units away from center along major axis</td>
<td></td>
</tr>
</tbody>
</table>
Example 1: Find the center, vertices, and foci of \[ \frac{(y+3)^2}{16} + \frac{(x-1)^2}{25} = 1. \]

Example 2: Find the equation for the ellipse that satisfies the given conditions.
Foci: \((-2, 1)\) and \((-2, 5)\), the major axis endpoints are \((-2, -1)\) and \((-2, 7)\)

Example 3: Prove that the graph of \(4x^2 + y^2 - 8x + 4y + 4 = 0\) is an ellipse. Then, find its center and foci.
8.3 Hyperbolas

8.3 HYPERBOLAS

Learning Targets 8.3
1. Write the equation of a hyperbola when given at least two of the important features of the graph.
2. Identify key features of a hyperbola when given an equation.
3. Sketch the graph of a hyperbola by hand including center and asymptotes.
4. Prove a general form equation is a hyperbola
5. Write the equations of the asymptotes of a hyperbola.

Hyperbola: The set of all points in a plane whose difference of distances from two fixed points is constant. The fixed points are called _____________. For the equations below, ____________ and ____________.

- Transverse Axis contains the center, vertices and foci.
- Conjugate Axis is perpendicular to the transverse axis.

Vertical Hyperbola: \[
\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1
\]

Horizontal Hyperbola: \[
\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1
\]

Example 1: List the center, vertices and foci of each hyperbola. Then, sketch the graph by hand.

a) \[
\frac{(x-1)^2}{9} - \frac{(y+3)^2}{25} = 1
\]

b) \[
\frac{y^2}{16} - \frac{(x+2)^2}{4} = 1
\]
In summary...

<table>
<thead>
<tr>
<th></th>
<th>Vertical Transverse Axis</th>
<th>Horizontal Transverse Axis</th>
<th>ALL Hyperbolas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices</td>
<td></td>
<td></td>
<td>“a” units away from center along transverse axis.</td>
</tr>
<tr>
<td>Foci</td>
<td></td>
<td></td>
<td>“c” units away from center along transverse axis.</td>
</tr>
<tr>
<td>Asymptotes</td>
<td></td>
<td></td>
<td>Drawn through the corners of box with dimensions 2a by 2b.</td>
</tr>
</tbody>
</table>

**Example 2:** Find the equation for the hyperbola that satisfies the given conditions.

Center at \((1, -4)\), focus \((1, -10)\) and vertex \((1, -1)\).

**Example 3:** Prove the graph of \(25y^2 - 9x^2 - 50y - 50x - 281 = 0\) is a hyperbola. Then, find its center and the equations of the asymptotes.