Pre-Calculus First Semester Review

NON CALCULATOR

[1.2] Find the domain. Express the answer in interval notation.

1. \( f(x) = \log_3 (2x + 5) \)  
2. \( f(x) = \frac{\sqrt{7-x}}{x+4} \)

[1.2] Prove algebraically whether the function is even, odd, or neither.

3. \( f(x) = 3x^3 - 2x \)  
4. \( f(x) = -2x^4 - 4x + 7\)

For question 5-10, find following:
(a) Identify the parent function.
(b) State the transformation rule or describe the transformation.
(c) Graph the function including key points and any asymptotes.

[1.5] 5. \( f(x) = -3\sqrt[2]{2x + 6} + 4 \)

[1.5] 6. \( f(x) = \log_4 (x - 2) \)

[1.5] 7. \( f(x) = 3^{x-2} + 1 \)

[1.5] 8. \( f(x) = \frac{1}{2x + 4} - 3 \)
9. \( f(x) = 3\sqrt{4-x} \)

10. \( f(x) = 3\sin(2x-\pi) - 1 \)

[1.3] Graph the piecewise-defined function. State whether the function is continuous or discontinuous at \( x = 0 \).

11. \( f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases} \)

12. \( f(x) = \begin{cases} -|x| & \text{if } x \leq 0 \\ \frac{2}{2} & \text{if } x > 0 \end{cases} \)

[2.3] For each function below…

a) Determine the degree.

b) Describe the end behavior using limit notation.

c) Find the zeros of the function with their multiplicities.

d) Sketch a graph of the function including zeros, multiplicities and end behavior.

13. \( f(x) = -4(x+2)^3(2x-3)^2 \)

14. \( f(x) = x^4 - 36x^2 \)
Find (if it exists) the a) equations of any horizontal or slant asymptote, b) equations of any vertical asymptote(s) and coordinates of any holes, c) x-intercept and y-intercept, and d) graph the function including additional points in each region of the domain OR using a sign chart.

15. \( g(x) = \frac{4x^2 - x - 5}{x^2 - 2x - 3} \)  

16. \( g(x) = \frac{-2x}{x^2 - x - 6} \)

17. Use the rational function below, along with the listed attributes, to graph the function. Include additional points in each region of the domain.

\[
f(x) = \frac{x^3 + x^2 - 9x - 9}{x^2 + 2x - 3} = \frac{(x + 3)(x - 3)(x + 1)}{(x + 3)(x - 1)}
\]

SA: \( y = x - 1 \)

VA: \( x = 1 \)

x-intercepts: (3, 0) and (–1, 0)

y-intercept: (0, 3)

18. Describe the end behavior of the rational function in question 40 using limit notation.

For questions 19 & 20, identify the letter of the graph below that best matches the given function.

19. \( f(x) = \frac{1}{2}x^{-5} \)

20. \( f(x) = 3x^{1/4} \)
Solve algebraically showing all steps. Check for extraneous roots. Write your answers in interval notation where appropriate.

[P3] 21. \(2(5 - 2y) - 3(1 - y) \geq y + 1\)  

[P5] 22. \(-3 \leq 1 - 2x < 7\)

[P5] 23. \(4x^2 - 7x + 5 = 0\)  

[P6] 24. \(|-x + 4| - 3 > 7\)

[2.6] 25. \((x - 4)^2 + 18 = 9\)  

[2.8] 26. \(\frac{3x}{x+1} + \frac{5}{x-2} = \frac{15}{x^2 - x - 2}\)

[P5] Solve by graphing. Note the procedure used.

27. \(3x - 2 = \sqrt{x + 4}\)  

28. \(0 = x^3 + x^2 - 5x + 3\)

[2.5] 29. Write in \(a + bi\) form: \(\frac{2 + 4i}{3 - 2i}\)
[2.6] Find a polynomial equation with the given zeros. a) Write the function as a product of linear and irreducible quadratic factors and b) Express function in standard form.

30. \(-1, 2 - i\)  
31. \(3, 4i\)

[2.6] Find the zeros of the function and write the function as a product of linear and irreducible quadratic factors all with real coefficients.

32. \(f(x) = x^3 - x^2 - x - 2\), given zero \(x = 2\)  
33. \(f(x) = x^4 + 3x^3 - 3x^2 + 3x - 4\), given zeros \(x = 1\) and \(x = -4\)

Solve questions 34-36 using as sign chart.

[P6] 34. \(12x^3 - 14x^2 - 6x \geq 0\)  
[2.9] 35. \(\frac{2x + 1}{x^2 + 2x - 3} \leq 0\)  
[2.9] 36. \(\frac{2}{x + 1} - \frac{3}{x - 5} > 0\)


a) \(2(5)^x = 26\)  
b) \(4 + 3e^{x-5} = 157\)  
c) \(\ln\left(\frac{x}{5}\right) = -0.2\)

d) \(5 = 21 - 2\log_3(x - 7)\)  
c) \(\log(x) + \log(x + 21) = 2\)  
d) \(\log_2(x - 1) - \log_2(2x - 3) = 3\)
38. Find all a) local maxima and minima and b) identify intervals on which the function is increasing and decreasing.

\[ f(x) = x^3 + 2x^2 - 6x \]

[1.2] Graph the function and tell whether or not it has a point of discontinuity at \( x = 0 \). If there is a discontinuity, tell whether it is removable or non-removable.

39. \( f(x) = \frac{|x|}{x} \)

40. \( h(x) = \frac{x^2 + x}{x} \)

Using the twelve basic parent functions provided in the box, list the equation of the function(s) that fit the description given.

| \( f(x) = x \) | \( f(x) = \ln x \) | \( f(x) = e^x \) | \( f(x) = x^2 \) | \( f(x) = |x| \) | \( f(x) = x^3 \) |
| \( f(x) = \sqrt{x} \) | \( f(x) = \frac{1}{x} \) | \( f(x) = \sin x \) | \( f(x) = \cos x \) | \( f(x) = \text{int}(x) \) | \( f(x) = \frac{1}{1 + e^{-x}} \) |

41. Bounded (3 functions).

42. Increasing on the entire domain (6 functions).

43. Even (3 functions).

48. Given: \( f(x) = (x - 4)^2 \), \( g(x) = 2x - 3 \) and \( h(x) = \sqrt{x + 5} \). Find and simplify the answer.

44. \( f(g(4)) \)

45. \( h(g(x)) \)

46. \( (g-f)(x) \)

47. \( (fg)(x) \)

48. Given: \( f(x) = x^3 + 2 \). Find \( f^{-1}(x) \).
[1.4] 49. **Verify** that \( f \) and \( g \) are inverses of each other. Use correct notation and show all algebraic steps.

\[
f(x) = 2x^2 + 8 \quad \text{and} \quad g(x) = \sqrt{\frac{x - 8}{2}}
\]

[P4] 50. Write the equation of a line passing through the point \((3, -4)\) that is a) parallel to \(5x - y = 7\), and b) perpendicular to \(5x - y = 7\).

[1.6] 51. The height of a right circular cylinder equals its diameter. Write the volume of the cylinder as a function of its radius.

[2.2] 52. Write the statement as a power function equation and answer the question. The electrical resistance of a wire varies directly as its length and inversely as the square of the diameter of the wire.

a) Write a model for this situation.

b) Suppose 50 mm of a wire of diameter 3 mm has a resistance of 8 \(\Omega\). Use this information to find the constant \(k\).

c) What is the resistance of 40 mm of the same type of wire if the diameter is 4 mm?

[2.2] 53. The table below gives the weight and pulse rate of selected mammals. Use the power regression equation to determine the pulse rate of a human weighing 12 pounds.

<table>
<thead>
<tr>
<th>Mammal</th>
<th>Body Weight</th>
<th>Pulse Rate (beats/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rat</td>
<td>0.2</td>
<td>420</td>
</tr>
<tr>
<td>Guinea Pig</td>
<td>0.3</td>
<td>300</td>
</tr>
<tr>
<td>Rabbit</td>
<td>2</td>
<td>205</td>
</tr>
<tr>
<td>Small Dog</td>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>Large Dog</td>
<td>30</td>
<td>85</td>
</tr>
<tr>
<td>Sheep</td>
<td>50</td>
<td>70</td>
</tr>
<tr>
<td>Human</td>
<td>70</td>
<td>72</td>
</tr>
</tbody>
</table>

**NOTE:** Any of the following regressions may be tested on the final: linear, quadratic, cubic, quartic, exponential, logarithmic, logistic, power, sinusoidal.
[2.1] 54. The Sweet Drip Beverage Company sells cans of soda in machines. The marketing director finds that sales average 26,000 cans per month when the cans sell for $0.50 each. For every $0.05 increase in the price, the sales per month drop by 1000 cans.

a) Write an equation to model the total revenue realized by Sweet Drip, where $x$ is the number of $0.05 increases in the price of a can of soda.

b) How much should Sweet Drip charge per can of soda to realize their maximum revenue?

[3.2] 55. Fruit flies are placed in a container with a banana and yeast plants. Suppose the fruit fly population after $t$ days is given by $P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$.

a) What is the maximum number of fruit flies the container can hold?

b) How many fruit flies were originally placed in the container?

c) How long does it take for the number of fruit flies to reach one-half of the maximum flies that the container can hold?

For questions 56-57, write a model for the situation. Be sure to clearly define your variables. Then use your model to answer the question. Solve algebraically AND graphically.

[3.2] 56. Shan invested $100 at 3.5% interest compounded monthly. Determine how long it will take for Shan to save up $700 assuming no additional deposits or withdrawals.

[3.2] 57. A radioactive isotope decays at a rate of 3% per day. A scientist has an initial amount of 50 g. Determine approximately how many days it will take for half the isotope to decay and then write a half-life model.
[3.2] 58. Use the TVM Solver:
   a) Find the monthly payment for a mortgage on a $275,000 house paid at the end of each month if the interest rate is 3.5% on a 30 year loan.

   b) After graduating from college and getting your first job, you decide to open an individual retirement account (IRA) using $500 you got for graduation. The account pays 6.2% interest at the beginning of each month and you have budgeted to invest $100 each month. If you retire in 25 years, what will IRA be worth?

[3.5] 59. The wind speed \( s \) (in miles per hour) near the center of a tornado can be modeled by \( s = 93 \log d + 65 \) where \( d \) is the distance (in miles) that the tornado travels.

   a) In 1925, a tornado traveled 220 miles through three states. Estimate the wind speed near the tornado’s center.

   b) An F5 tornado has wind speeds of more than 300 mph, estimate the distance an F5 tornado with wind speed of 310 mph will travel.

[4.8] 60. When a spaceship is fired into orbit from a site such as Cape Canaveral, which is not on the equator, it goes into an orbit that takes it alternately north and south of the equator. Its distance from the equator can be approximately modeled by a sinusoid function.

   Suppose that the spaceship is fired into orbit from Cape Canaveral. Ten minutes after it leaves the Cape, it reaches its farthest distance north of the equator, 4000 kilometers. Half a cycle later it reaches its farthest distance south of the equator (on the other side of the Earth), also 4000 kilometers. The spaceship completes an orbit once every 90 minutes.

   Let \( y \) be the number of kilometers the spaceship is north of the equator (you may consider south of the equator to be negative). Let \( x \) be the number of minutes that have elapsed since liftoff.

   a) Sketch a complete cycle of the graph of distance versus time.

   b) Write the sinusoidal model.

   c) Use your equation to predict the distance of the spaceship from the equator when \( x = 41 \) and \( x = 163 \) mins.

   d) Find the number of kilometers Cape Canaveral is from the equator by calculating \( y \) when \( x = 0 \).

Since we just finished Unit 4: Conic Sections, there are no review problems included on these sections but Conic sections will be on the final. Go back and use your Unit 4 Review to study.