1. What is the relationship between position, velocity, and acceleration?

Suppose position is \( s(t) \).

Velocity is derivative of position or \( v(t) = s'(t) \) and acceleration is derivative of velocity or \( a(t) = v'(t) = s''(t) \).

2. Once again trying to blow up earth because it interferes with his view of Venus, Marvin the Martian lands on the moon. Bugs Bunny, as always, interferes with his plan. Chasing Bugs, Marvin fires a warning shot straight up into the air with his Acme Disintegration Pistol. The height (in feet) after \( t \) seconds of the shot is given by

\[ s(t) = -2.66t^2 + 135t + 3. \]

a) Find the velocity and acceleration as functions of time.

(What is the meaning of the acceleration function?)

\[ v(t) = s'(t) = -5.32t + 135 \]

\[ a(t) = v'(t) = -5.32 \quad \text{This means the pull of gravity on the moon is} \ -5.32 \text{ ft/sec approx to that of Earth.} \]

b) What is the position of the shot when the velocity is 0?

\[ v(t) = 0 \quad \therefore -5.32t + 135 = 0 \]

\[ -5.32t = -135 \]

\[ t = 25.37593588... \]

\[ v(t) = 0 \quad \text{when} \ t = 25.375 \text{ sec.} \]

3. Fill in the blanks.

a) When the ______velocity_______ is positive, the object is moving in a positive direction.

b) An object is ______slowing down_______ when the velocity and acceleration have different signs.

c) An object is stopped when ______velocity_______ is zero.

d) Speed is always positive because it is the ______absolute value_______ of velocity.

4. A bug begins to crawl up a vertical wire at time \( t = 0 \). The velocity, \( v \), of the bug at time \( t \), \( 0 \leq t \leq 8 \) is given by the function whose graph is shown below.

a) At what value of \( t \) does the bug change direction? Justify your response.

The bug changes direction at \( t = 6 \) because \( v(t) \) changed from pos to neg.

b) During which time intervals in the bug slowing down? Justify your response.

The bug is slowing down on (4, 6) because \( v(t) > 0 \) and \( a(t) < 0 \). The bug also slows down on (7, 8) because \( v(t) < 0 \) and \( a(t) > 0 \).
5. The figure graphed below shows the velocity of a particle moving along a coordinate line. Justify each response.

a) When is the particle moving right?
The particle moves right when
\[ v(t) > 0 \implies \text{moves right on } (4,10) \]

b) When is the particle moving left?
The particle moves left when
\[ v(t) < 0 \implies \text{moves left on } (0,4) \]

c) When is the particle stopped?
The particle is stopped at \( t = 0 \) and \( t = 4 \)
d) When is the particle speeding up?
Particle speeds up on \((0,1)\) b/c \( v(t) > 0 \) and \( a(t) > 0 \).
Also speeds up on \((4,5)\) b/c \( v(t) > 0 \) and \( a(t) > 0 \).
e) When does the particle change directions?
Particle changed directions at \( t = 4 \)
   \( \text{b/c velocity changes from neg to pos.} \)
f) When is the particle slowing down?
Particle slows down on \((3,4)\) b/c \( v(t) < 0 \) and \( a(t) < 0 \).
   Also slows down on \((5,7)\) b/c \( v(t) > 0 \) and \( a(t) < 0 \).
g) What is the particle’s greatest speed and when is the particle moving at its greatest speed?
Speed = \( |v(t)| \) Greatest speed is 4 at
\[ t = 5 \] and on \((1,3)\) b/c \( |v(t)| \) is largest.
i) When is the particle’s acceleration negative?
Particles acc. is neg on \((0,1)\) b/c \((5,7)\) b/c \((9,10)\)
   \( \text{b/c the slope of velocity is neg.} \)

6. Fill in the blanks with correct mathematical notation.

a) If you want the average velocity of a particle on the interval \([2, 5]\), you must find
\[ \frac{S(5) - S(2)}{5 - 2} \] when \( S(t) = \text{pos} \)

b) If you want the velocity of a particle at \( t = 4 \), you must find
\[ S'(4) = v(4). \]

\[ \text{Instant slope = derivative} \]

\[ \text{IROC = Instant Slope = derivative} \]

7. Velocity is the rate of change of position. If the position of a particle on the \( x \)-axis at time \( t \) is given by \( -5t^2 \), then what is the average velocity of the particle for \( 0 \leq t \leq 3 \)?
\[ x(t) = -5t^2 \]
\[ \text{Avg. vel.} = \frac{x(3) - x(0)}{3 - 0} = -15 \]

8. A particle moves along the \( x \)-axis so that its position at time \( t \) is given by \( x(t) = t^2 - 6t + 5 \). For what value of \( t \) is the velocity of the particle zero?
\[ v(t) = x'(t) = 2t - 6 \]
\[ 0 = 2t - 6 \]
\[ t = 3 \]
\[ \therefore \text{At } t = 3, \ v(t) = 0. \]
9. Fill in the blanks with correct mathematical notation.

a) If you want the average acceleration of a particle on the interval [1, 3], you must find \[ \frac{v(3) - v(1)}{3-1} \] where \( v(t) \) = velocity

b) If you want the acceleration of a particle at \( t = 8 \), you must find \[ a(8) = v'(8) = a''(8) \]

10. Rocket \( A \) has a positive velocity \( v(t) \) after being launched upward from an initial height of 0 feet at time \( t = 0 \) seconds. The velocity of the rocket is recorded for selected values of \( t \) over the interval \( 0 \leq t \leq 80 \) seconds as shown in the table below.

<table>
<thead>
<tr>
<th>( t ) (sec)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) ) (ft/sec)</td>
<td>5</td>
<td>14</td>
<td>22</td>
<td>29</td>
<td>35</td>
<td>40</td>
<td>44</td>
<td>47</td>
<td>49</td>
</tr>
</tbody>
</table>

a) Find the average acceleration of Rocket \( A \) over the time interval \( 0 \leq t \leq 80 \) seconds. Indicate units of measure.

\[ \frac{\Delta v(t)}{\Delta t} = \frac{v(80) - v(0)}{80-0} = \frac{49-5}{80-0} = \frac{44}{20} \text{ ft/sec} = 2.2 \text{ ft/sec}^2 \]

b) Using the data in the table, find an estimate for \( v'(35) \). Indicate units of measure.

\[ \lim_{h \to 0} \frac{f(x+h)-f(x-h)}{2h} \]

11. A particle moves along the \( x \)-axis so that its position at any time \( t \geq 0 \) is given by the function \( x(t) = t^3 - 12t + 1 \), where \( x \) is measured in feet and \( t \) is measured in seconds. Justify each response and indicate units of measure when appropriate.

a) Find the displacement during the first 3 seconds.

\[ x(3) - x(0) = (3^3 - 12(3) + 1) - (0) = 27 - 36 = -9 \text{ ft} \] This means particle moved left 9 feet.

b) Find the average velocity during the first 3 seconds.

\[ \frac{\Delta \text{pos}}{\Delta t} = \frac{x(3) - x(0)}{3-0} = \frac{-9}{3} = -3 \text{ ft/sec} \]

c) Find the instantaneous velocity at \( t = 3 \) seconds.

\[ v(t) = x'(t) = 3t^2 - 12 \]

\[ v(3) = 3(3)^2 - 12 = 15 \text{ ft/sec} \]

d) Find the acceleration when \( t = 3 \) seconds.

\[ a(t) = v'(t) = 6t \]

\[ a(3) = 6(3) = 18 \text{ ft/sec}^2 \]

e) When is the particle moving left?

Particle moves left on \((0, 2)\) blc \( v(t) < 0 \).

g) When is the particle speeding up?

Particle speeds up when \( t > 2 \sec \) blc \( v(t) > 0 \) and \( a(t) > 0 \).
12. [Calculator] A particle moves along a line so that at time \( t \), \( 0 \leq t \leq \pi \), its position is given by
\[
s(t) = -4 \cos t - \frac{t^2}{2} + 10.\]
What is the velocity of the particle when its acceleration is zero?
\[
v(t) = s'(t) = -4(-\sin t) - \frac{1}{2}(2t) + 0
\]
\[
v(t) = 4 \sin t - t
\]
\[
v(1.318...) = 2.555
\]
Find \( t \) for \( a(t) = 0 \) on \( 0 \leq t \leq \pi \)
\[
4 \cos t - 1 = 0
\]
\[
\cos t = \frac{1}{4}
\]
\[
t = 1.318...
\]

13. [No Calculator] A spring is bobbing up and down. Its position at any time \( t \geq 0 \) is given by \( s(t) = -4 \sin t \).

a) What is the initial position of the spring?
\[
t = 0; s(0) = -4 \sin (0) = 0
\]
b) Which way is the particle moving to start? Justify your response.
\[
\text{We need rate of change of position...}
\]
\[
v(t) = s'(t) = -4 \cos t
\]
\[
v(0) = -4 \cos (0) = -4
\]
The spring moves down at \( t = 0 \) b/c \( v(0) < 0 \)
c) At \( t = \frac{\pi}{2} \), is the spring moving up or down? Justify your response.
\[
v(\frac{\pi}{2}) = -4 \cos (\frac{\pi}{2}) = -4(-\frac{\sqrt{2}}{2}) = 2\sqrt{2}
\]
\[
\text{Since } v(\frac{\pi}{2}) > 0, \text{ the spring moves up when } t = \frac{\pi}{2}.
\]
d) Is the spring speeding up or slowing down at \( t = \frac{\pi}{4} \)? Justify your response.
\[
a(t) = -4(-\sin t) + 4 \sin t
\]
\[
a(\frac{\pi}{4}) = 4 \sin (\frac{\pi}{4}) = 4 (-\frac{\sqrt{2}}{2}) = -2\sqrt{2}
\]
The spring is slowing down at \( t = \frac{\pi}{4} \) b/c \( v(\frac{\pi}{4}) < 0 \) and \( a(\frac{\pi}{4}) < 0 \).

14. [Calculator Required] A body is moving in simple harmonic motion (up/down) with position \( s(t) = 3 + \cos t \), where \( 0 \leq t < 2\pi \).

a) Find \( v(t) \), the velocity function.
\[
v(t) = s'(t) = -\sin t
\]
b) Find the zeros of \( v(t) \).
\[
0 = -\sin t
\]
\[
0 = \sin t
\]
\[
\text{On } 0 \leq t < 2\pi, \text{ the zeros at } t = 0 \text{ and } t = \pi
\]
c) Find \( a(t) \), the acceleration function.
\[
a(t) = v'(t) = -\cos t
\]
d) Find the zeros of \( a(t) \).
\[
0 = -\cos t
\]
\[
0 = \cos t
\]
\[
\text{On } 0 \leq t < 2\pi, \text{ the zeros at } t = \frac{\pi}{2} \text{ and } t = \frac{3\pi}{2}
\]
e) When is the object stopped? Justify your response.
\[
\text{Object is stopped at } t = 0 \text{ and } t = \pi \text{ b/c } v(t) = 0.
\]
f) When does the object change direction? Justify your response.
\[
v(t) = -\sin t
\]
\[
The \text{ object changed directions at } t = \pi
\]
\[
\text{b/c } v(t) < 0 \text{ when } t < \pi \text{ and } v(t) > 0 \text{ when } t > \pi,
\]
g) When does the object speed up? Justify your response.
\[
a(t) = -\cos t
\]
\[
The \text{ object speeds up on } (0, \frac{\pi}{2}) \text{ b/c } v(t) > 0 \text{ and } a(t) < 0.
\]
\[
The \text{ object also speeds up on } (\pi, \frac{3\pi}{2}) \text{ b/c } v(t) > 0 \text{ and } a(t) > 0.